Word Problem-Solving Instruction in Inclusive Third-Grade Mathematics Classrooms

CYNTHIA C. GRIFFIN
University of Florida

ASHA K. JITENDRA
University of Minnesota

ABSTRACT. The authors examined the effectiveness of strategy instruction taught by general educators in mixed-ability classrooms. Specifically, the authors compared the mathematical word problem-solving performance and computational skills of students who received schema-based instruction (SBI) with students who received general strategy instruction (GSI). Participants were 60 3rd-grade student participants randomly assigned to treatment conditions. Teachers pretested and posttested participants with mathematical problem-solving and computation tests, repeatedly measuring their progress on word problem solving across the 18-week intervention. Both SBI and GSI conditions improved word problem-solving and computation skills. Further, results show a significant difference between groups on the word problem-solving progress measure at Time 1, favoring the SBI group. However, this differential effect did not persist over time. The authors discuss implications for future research and practice.

Keywords: elementary mathematics instruction, mathematics word problem solving, mixed-ability classrooms, strategy instruction

The *Principles and Standards for School Mathematics* by the National Council of Teachers of Mathematics (NCTM; 2000) and the report "Adding It Up: Helping Children Learn Mathematics" by the National Research Council (NRC; 2001) have articulated a shift in emphasis from procedural knowledge, such as learning how to perform or apply algorithms, to conceptual understanding in mathematics instruction and assessment (Goldsmith & Mark, 1999; Hiebert et al., 1996; Romberg, Carpenter, & Kwako, 2005). However, with the recent publication of the *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics*, the NCTM (2006) revisited the role of procedural knowledge in mathematics instruction and reinforced its benefits when taught in classroom contexts that "promote problem solving, reasoning, communication, making connections, and designing and analyzing representations" (p. 15).

Problem solving, in particular, is a key theme in *Standards* (NCTM, 2000) and *Focal Points* (NCTM, 2006). Learning how to solve story problems involves knowledge about semantic structure and mathematical relations as well as knowledge of basic numerical skills and strategies. Yet, story problems pose difficulties for many students because of the complexity of the solution process (Jonassen, 2003; Lucangeli, Tressoldi, & Cendron, 1998; Schurter, 2002). Because problem solving, as a process, is more complex than simply extracting numbers from a story situation to solve an equation, researchers and educators must afford attention to the design of problem-solving instruction to enhance student learning. Unfortunately, traditional mathematics textbooks typically do not provide the kind of instruction recommended by the NCTM. In a study that evaluated five third-grade mathematics textbooks with regard to their adherence to *Standards*, Jitendra et al. (2005) found that the textbooks inadequately addressed them. In particular, opportunities for reasoning and making connections were present in less than half the instances in these textbooks.

Providing classroom opportunities that emphasize mathematical thinking and reasoning is critical for successful problem solving. However, these skills are not well addressed in traditional mathematics textbooks for several reasons. On the one hand, when the same procedure (e.g., addition) is used to solve all problems on a page or in a chapter, students do not have the opportunity to discriminate among problems that require different solutions. On the other hand, teaching students to use key words (e.g., *in all* suggests addition, *left* suggests subtraction, *share* suggests division; Lester, Garofalo, & Kroll, 1989, p. 84) is misleading, because "many problems do not have key words," and "key words send a terribly wrong message about doing math" (Van de Walle, 2007, p. 152). An overreliance on key words does not develop conceptual understanding, because this approach ignores the meaning and structure of

Address correspondence to Cynthia C. Griffin, University of Florida, Department of Special Education, G315 Norman Hall, P.O. Box 117050, Gainesville, FL 32611, USA. (E-mail: ccgriffin@coe.ufl.edu)

the problem and fails to develop reasoning and sense making of problem situations (Van de Walle).

A growing body of evidence suggests that strategy instruction in mathematics is a powerful approach to helping students learn and retain not only basic facts but also higher order skills, like problem solving (e.g., Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Pressley & Hilden, 2006). Effective instruction fosters the development of a variety of strategies and also supports students' gradual shift to the use of more efficient retrieval and reasoning strategies (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Siegler, 2005). Instructional strategies that researchers have found to be consistently effective for teaching students who experience learning difficulties in mathematics include depicting problems visually and graphically, teaching math concepts and principles by using explicit instruction, and using peer-assisted learning activities during mathematics instruction (Baker, Gersten, & Lee, 2002; L. S. Fuchs, Fuchs, Yazdian, & Powell, 2002; Jitendra, Griffin, Deatline-Buchman, et al., 2007; Kroesbergen, Van Luit, & Maas, 2004; Van Garderen & Montague, 2003). In addition to these strategies, interventions that provide feedback to teachers and students regarding student performance in mathematics and discussions of student successes with parents were found to be effective (Baker et al.). At the same time, the NCTM's *Standards* and some researchers have highlighted the importance of students' learning to apply and adapt a variety of strategies to solve mathematical problems and moving students beyond "routine expertise" to the development of "adaptive expertise" (Torbeyns, Verschaffel, & Ghesquière, 2005, p. 1; see also Kilpatrick, Swafford, & Findell, 2001). Thus, recent editions of popular mathematics textbooks recommend that teachers use a multitude of strategies to help students approach problem solving in a flexible manner.

Typically, mathematics textbooks include general strategy instruction (GSI) that involves the use of heuristic and multiple strategies based on Pólya's (1945/1990) seminal principles for problem solving (Lopez-Real, 2006). Pólya's four-step problem-solving model includes the following stages: (a) understand the problem, (b) devise a plan, (c) carry out the plan, and (d) look back and reflect. Each stage is further defined by the use of questions and explanations. For example, to understand the problem, supporting questions include the following: Do you understand all the words used in stating the problem? and What are you asked to find or show? Pólya suggested that there are many ways to solve problems and that students should learn how to choose appropriate strategies, such as working backward, using a formula, and looking for a pattern.

However, GSI has come under scrutiny for several reasons. First, the plan step in GSI involves a general approach to the problem-solving task. For example, a common visual representation strategy in GSI—draw a diagram—is at a general level and may not necessarily emphasize the importance of depicting the relations between elements in the problem, which is necessary for successful problem solving (Hegarty & Kozhevnikov, 1999). Second, although multiple strategies are perceived to have the potential for promoting mathematics learning, the following questions remain unanswered (Woodward, 2006): Do these strategies have adequate instructional support to be effective with young children? Does exposing all students to multiple strategies and processes lead to successful problem solving? Consequently, in the present study, we examined the differential effects of two types of strategy instruction: schema-based instruction (SBI) and GSI involving multiple strategies typically found in mathematics textbooks (e.g., use objects, draw a diagram, write a number sentence, use data from a graph). In the next section, we provide background information that guided the development of our schema-based strategy instruction and a review of related research.

Theoretical Framework

Schema theories of cognitive psychology are helpful in understanding and assessing childrens' solution of word problems (Briars & Larkin, 1984; Carpenter & Moser, 1984; Kintsch & Greeno, 1985; Riley, Greeno, & Heller, 1983). A number of research studies evince that semantic structure in word problems has much more influence than the arithmetic syntax on children's problem-solving strategies (Carpenter, Hiebert, & Moser, 1983; Carpenter & Moser; Fuson, Carroll, & Landis, 1996; García, Jiménez, & Hess, 2006; Vergnaud, 1997). *Semantic relations* refer to "conceptual knowledge about increases, decreases, combinations, and comparisons involving sets of objects" (Riley et al., p. 159). Additive problem structures in arithmetic that are characteristic of most addition and subtraction word problems in elementary mathematics textbooks include the problem types of change, combine, and compare (Carpenter & Moser).

A *change* problem involves an increase or decrease of an initial quantity to result in a new quantity. The three sets of information in a change problem are the beginning, change, and ending. In a *combine* problem, two distinct groups or subsets (parts) combine to form a new group (whole) or set. The relation between a particular set and its subsets is static. A *compare* problem involves the comparison of two disjoint sets (compared and referent), and the relation between the two sets is static. The three sets of information in a compare problem are the compared, referent, and difference sets. For each problem type, the position of the unknown in these problems may be any one of the three aforementioned sets.

The emphasis on the semantic structure and problem representation in SBI serves to enhance mathematical problem solving. *Schemata* are domain- or context-specific knowledge structures that serve the function of knowledge organization by allowing the learner to categorize various problem types to determine the most appropriate actions needed to solve the problem (Chen, 1999; Kalyuga, Chandler, Tuovinen,

& Sweller, 2001). According to Marshall (1995), schemata "capture both the patterns of relationships as well as their linkages to operations" (p. 67).

Building on the work of Marshall (1995), Mayer (1999), and Riley et al. (1983), we subsequently describe a problem-solving model that is the basis of our schema-based treatment. Essential elements of the model are four separate but interrelated problem-solving procedural steps. The four steps are problem schema identification, representation, planning, and solution. The corresponding conceptual knowledge for each step includes schema knowledge, elaboration knowledge, strategic knowledge, and execution knowledge.

Schema knowledge and problem schema identification. A critical function of schema-based instruction is pattern or schema recognition, which involves schematic knowledge for problem identification (Mayer, 1999). Recognition of a problem schema (e.g., compare) is facilitated when the basic semantic relations (e.g., "Howard read 4 more books than Tony") among the various problem features are evident. Problem schema recognition involves simultaneous processing of the various problem features (e.g., compared, referent, difference). Different problem schemata (change, group, compare) have their own distinct core features.

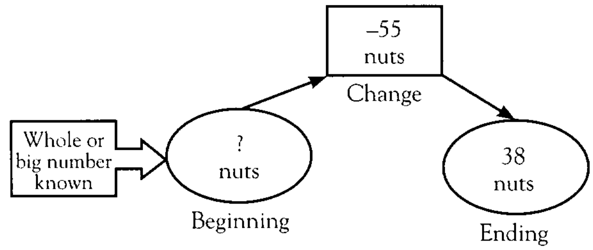
Elaboration knowledge and problem representation. The second step involves developing a schematic diagram or template that corresponds with the representation of the problem identified in the first step (for schematic diagrams, see Figure 1). Specifically, this step entails elaborating on the "main features of the situation or event around which the schema was developed" (Marshall, 1995, p. 40). Understanding is demonstrated by how the learner maps the details of the problem onto the schema diagram. At this time, all irrelevant information in the problem is discarded, and representation of the problem is based on available schema elaboration knowledge.

Strategic knowledge and problem–solution planning. The third step refers to planning, which involves (a) setting up goals and subgoals, (b) selecting the appropriate operation (e.g., addition), and (c) writing the math sentence or equation. A problem solver may successfully identify and elaborate on a specific schema in a problem but may not demonstrate strategic knowledge to plan for the solution. Planning may not be necessarily straightforward for multistep problems. Understanding mathematical situations that require the application of arithmetic conceptual knowledge (e.g., subtraction is an appropriate operation to solve for the part) is crucial to problem solution.

Execution knowledge and problem solution. The last step of problem solving is to carry out the plan. Execution knowledge consists of techniques that lead to problem solution, such as performing a skill or following an algorithm. Such knowledge may be shared among many schemata. For example, solving additive problem structures such as the change, group, and compare problems requires carrying out the addition or subtraction operation, because these

A

A squirrel made a pile of nuts. It carried away 55 nuts up to its nest. Now, there are 38 nuts in the pile. How many nuts were in the pile at the beginning?



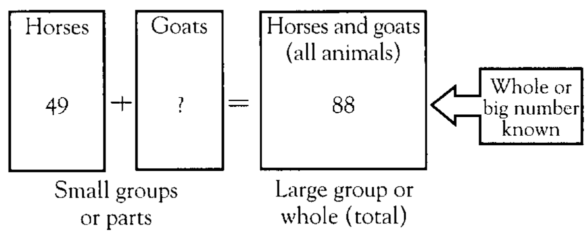
? – 55 = 38

$$\begin{array}{r} 55 \\ +\ 38 \\ \hline 93 \end{array}$$ nuts

Answer: 93 nuts

B

Farmer Joe has 88 animals on his farm. He only has horses and goats. There are 49 horses on the farm. How many goats are on the farm?



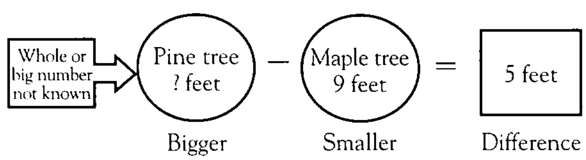
49 + ? = 88

$$\begin{array}{r} 88 \\ -\ 49 \\ \hline 39 \end{array}$$ goats

Answer: 39 goats

C

Craig saw a pine tree in the forest. Later, he saw a maple tree that was 9 feet tall. The maple tree was 5 feet shorter than the pine tree. How tall is the pine tree?



? – 9 = 5

$$\begin{array}{r} 9 \\ +\ 5 \\ \hline 14 \end{array}$$ feet

Answer: 14 feet

FIGURE 1. Solving Change (A), Group (B), and Compare (C) Problems. Change diagram adapted from S. P. Marshall (1995, p. 133). Copyright © 1995 Cambridge University Press. Adapted with permission.

problems are based on the part–part–whole mathematical concept. The difference between planning and solution is that the former focuses on a particular choice and order of operation, whereas the latter implements the plan.

Relevant Research

The present study focused on the solving of addition and subtraction word problems of third-grade students in heterogeneous mathematics classrooms. Because problem solving—particularly problem schema identification and representation—is difficult for many students (e.g., Mayer & Hegarty, 1996), it is important to teach them to construct a coherent model to represent the situation that the text presents and then to plan a solution on the basis of the model (Hegarty, Mayer, & Monk, 1995). Consequently, many research studies in the past decade have emphasized a model of schema-mediated problem-solving instruction. These studies have focused on (a) SBI that used either number line diagrams for understanding the semantic structure of compare word problems (e.g., Lewis, 1989; Zawaiza & Gerber, 1993) or schematic diagrams for solving a range of word problems (e.g., Fuson & Willis, 1989; Jitendra, Griffin, Haria, et al., 2007; Jitendra et al., 1998; Willis & Fuson, 1988; Xin, Jitendra, & Deatline-Buchman, 2005), (b) schema-induction instruction (Chen, 1999; Quilici & Mayer, 1996), (c) schema-broadening instruction with explicit instruction for supporting transfer by focus on similar problem types (e.g., L. S. Fuchs et al., 2006; L. S. Fuchs et al., 2003a), and (d) schema-broadening instruction with metacognitive instruction (L. S. Fuchs et al., 2003b; Hutchinson, 1993). Collectively, this research shows that the effects for SBI are positive.

Even though schema training that focuses on priming the problem structure has been effective with college students (Quilici & Mayer, 1996) and older elementary school children (Chen, 1999), with regard to inducing problem–solution accuracy, this research has not addressed the benefits of such training for younger children and children at risk for mathematics difficulties. These children may need direct instruction and structured practice rather than independent exploration of variant procedural features of classic water jar problems, as in the Chen study, or independent sorting of problems according to problem types (Quilici & Mayer). Therefore, the problem-solving research by L. S. Fuchs and colleagues (e.g., L. S. Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; L. S. Fuchs et al., 2006; L. S. Fuchs et al., 2003a; L. S. Fuchs et al., 2003b) with third-grade students (low, average, and high achievers) is important. Their line of research has shown that explicit instruction in problem–solution and schema-broadening instruction as well as self-regulated learning strategies is necessary to enhance mathematical problem-solving performance. However, none of these studies examined the long-term effects of schema-mediated instruction or the benefits of visual representation techniques, such as those involving diagrams, that are helpful scaffolds in organizing information in the problem, thereby reducing the level of cognitive load associated with the problem-solving task (Pawley, Ayres, Cooper, & Sweller, 2005; Sweller & Low, 1992; Sweller, Van Merrienboer, & Paas, 1998).

Although the aforementioned studies have highlighted the utility of schema-mediated instruction, other treatment outcome studies have indicated that the use of SBI that emphasizes visual representations can be associated with success in problem solving (Stylianou & Silver, 2004). For example, Lewis (1989) investigated the effects of diagram training that included problem translation and integration training using a number line on college students' ability to accurately represent compare problems involving both additive and multiplicative problem structures. On the training items, results indicated greater pretest-to-posttest gains for the diagram-training group when compared with the group that received only translation training or the no-training control group. Similarly, the effects for transfer items revealed that the diagram group achieved greater gains than did either the statement group or the control group.

Zawaiza and Gerber's (1993) study extended the work of Lewis (1989) to postsecondary students with learning disabilities. The visual representation approach was moderately effective in helping students to solve compare word problems. Students with learning disabilities in the diagram group improved significantly on reversing their errors so that their performance resembled that of their math-competent peers. In contrast to this research with older students and the focus on compare problems only, the studies by Fuson and Willis (1989) and Willis and Fuson (1988) with high- and average-achieving second-grade students have examined whether an approach that focused not only on the solution procedures but also primarily on the semantic situation in change, combine, and compare problems can be effective when either the researcher or the classroom teachers gave the instruction. These two studies provided initial support for the use of schematic drawings to help students to successfully solve addition and subtraction word problems involving the semantic structure of the problem situations.

In addition, Jitendra and colleagues used schematic diagrams as part of SBI to teach elementary and middle school students with disabilities and those at risk for mathematics difficulties (e.g., Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Griffin, Deatline-Buchman, et al., 2007; Jitendra et al., 1998; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999; Xin et al., 2005) to solve a range of word problems in mathematics textbooks. Specifically, two studies' researchers compared the effectiveness of SBI with traditional textbook instruction using randomized controlled trials (Jitendra et al., 1998; Xin et al.). Results of these studies showed moderate to large effects on immediate, maintenance, and generalization posttests. However, this research on SBI using schematic diagrams has not addressed its effectiveness in comparison with alternative approaches in meeting the needs of all learners in general education settings when teachers, rather than researchers, gave problem-solving instruction.

However, more recently, Jitendra, Griffin, Haria, et al. (2007) investigated the differential effects of SBI and GSI in promoting mathematical problem solving, computation skills, and mathematics achievement of low-performing third-grade students when taught by classroom teachers. In this study, SBI included explicit instruction in using schematic diagrams to depict word problems visually prior to solving them. Results revealed that SBI was more effective than GSI in enhancing students' mathematical word problem-solving skills at posttest and maintenance. Further, the SBI groups' performance exceeded that of the GSI group on the Pennsylvania System of School Assessment measure. On the computation test, both groups made gains over time. Jitendra, Griffin, Haria, et al. found SBI along with schematic diagrams to be a powerful instructional approach for low-achieving students. However, the effects of SBI on students taught in mixed-ability (i.e., high, average, low achieving) classrooms have yet to be explored.

Purposes of the Present Study

The primary research question that we addressed in the present study was whether students benefit from either (a) SBI that focuses on the underlying problem structure by analyzing the problem schema and representing the problem by schematic diagrams or (b) GSI that incorporates multiple strategies. The GSI incorporated Pólya's (1945/1990) general instructional heuristic that are typically in mathematics textbooks (read and understand, plan, solve, and check) and included multiple representational and solution strategies (e.g., use manipulatives, use a table, write a number sentence). In light of the emphasis in *Standards* on equity (i.e., high expectations and strong support for all students) and the emphasis in the No Child Left Behind Act of 2001 on measuring educational outcomes for all students, it is important to examine the differential effects of SBI and GSI in the context of mixed-ability, general education mathematics classrooms. We expected that students in both conditions would improve in problem solving. However, we hypothesized that SBI would lead to better problem-solving performance than GSI because problem representation in SBI is scaffolded by diagrams that emphasize a "schematic imagery (i.e., representing the spatial relationships between objects and imagining spatial transformations)" (Hegarty & Kozhevnikov, 1999, p. 685) and that are known to be effective in mathematical problem solving.

A second question in the present study examined whether the strategy effects would persist over time. Unlike the Jitendra et al. (1998) and Jitendra, Griffin, Haria, et al. (2007) studies that assessed maintenance effects 1–2 or 6 weeks later, the present study extended it to 12 weeks following the termination of the intervention. A third question was whether the changes in word problem-solving performance over time during the intervention phase were similar for the two conditions. We expected students who received SBI to show immediate improvement during the intervention because SBI and its focus on the underlying mathematical problem structures help to connect the different mathematical operations (i.e., addition and subtraction). A possible disadvantage of the GSI is that addition word problems are taught first, preceding subtraction word problems, so that problem-solving strategies in GSI focus on these mathematical operations sequentially rather than in a way that enables comparisons and opportunities to discriminate between the two operations, as in SBI. Finally, we assessed the influence of word problem-solving instruction on the development of computational skills. We hypothesized that students' computational skills in both groups would improve over the course of the study as a result of their receiving word problem-solving instruction, as in Jitendra, Griffin, Haria, et al.'s study.

Method

The study used a between-subjects, experimental, pretest-to-posttest-to-delayed-posttest group design to investigate the effects of word problem-solving strategy instruction.

Participants

Participants were 60 students (30 boys [50%], 30 girls [50%]; M age = 107.65 months; *SD* = 4.69 months; age range = 100–121 months) from three classrooms attending third grade in an elementary school in a college town in the southeastern United States. Regarding participants' race and ethnicity, 37 (61.7%) were White; 14 (23.3%) were African American, and 9 (15%) were Hispanic American.

Students were rank ordered and then matched on their scores on the Mathematical Problem Solving subtest of the Stanford Achievement Test–9 (SAT-9 MPS; Harcourt Brace & Company, 1996). Next, each member of a matched student pair was randomly assigned to either the intervention or comparison condition. Further, students in each condition were randomly assigned to two instructional groups of 15 students each. In short, four instructional groups resulted from mixing students from the three classrooms. Two groups of students received SBI, and the other two groups participated in the comparison condition, receiving GSI.

Table 1 provides student demographic data for the SBI and GSI groups. A one-way analysis of variance (ANOVA) indicated no statistically significant differences between the groups on chronological age, $F(1, 58) = .02$, *ns*; on the SAT-9 MPS, $F(1, 58) = 0.00$, *ns*; or on the SAT-9 Mathematical Procedures Subtest (SAT-9 MP; Harcourt Brace & Company, 1996), $F(1, 58) = 0.22$, *ns*. Chi-square analyses also revealed no statistically significant between-groups differences on gender, $\chi^2(1, N = 60) = 0.61$, *ns*, or ethnicity, $\chi^2(2, N = 60) = 0.38$, *ns*.

TABLE 1. Student Demographics

Variable	SBI *M*	SBI *SD*	SBI *n*	SBI %	GSI *M*	GSI *SD*	GSI *n*	GSI %	$F(1, 58)$	χ^2
Age	107.73	5.06			107.57	4.38			0.02	
Gender										
Male			16	53.33			14	46.67		0.61
Female			14	46.67			16	53.33		
Ethnicity										
White			21	70.00			16	53.33		0.38
African American			3	10.00			6	20.00		
Hispanic American			6	20.00			8	26.67		
Learning disability			3	10.00			2	6.67		
SAT-9										
Problem solving	25.77	8.70			25.80	8.19			0.00	
Procedures	12.10	5.67			11.43	5.26			0.22	

Note. SBI = schema-based instruction; GSI = general strategy instruction; SAT-9 = Stanford Achievement Test–9.

Four female teachers (the three classroom teachers and one special education teacher who participated in the study) were randomly assigned to the two conditions and provided all instruction in the study. Teachers were White and had teaching experience of 2, 6, 10, and 20 years. All teachers were certified in elementary education, and one teacher was additionally certified in special education. To control for teacher effects, the four teachers switched halfway through the study to teach the other condition. These teachers attended two 2-hr in-school workshops (prior to and at the middle of the intervention) on implementing the treatments. The workshops provided a rationale for and content on the treatment programs. We modeled and discussed the scripts as the teachers reviewed them to familiarize themselves with the instructional procedures. Teachers were encouraged to study the scripts—rather than to read the scripts verbatim—to understand the instructional procedures for implementing the assigned strategy.

Materials

We used several one- and two-step word problems derived from five third-grade mathematics textbooks to teach word problem solving using the assigned strategy instruction (described later). In addition, story situations involving the three problem types that did not include unknown information were developed for use during the initial phase of SBI. Teacher materials for the two conditions included scripted lessons to ensure consistency of information. In addition, teacher materials in the SBI condition consisted of posters of schematic diagrams for the three problem types as well as story and word problem-solving checklists. Student materials included worksheets with schematic diagrams and word problem-solving checklists. For the GSI condition, teacher materials included a poster of word problem-solving steps, whereas student materials consisted of manipulatives (e.g., counters) and problem-solving worksheets.

Procedures

Table 2 presents a comparison of the two conditions. Students in both SBI and GSI conditions received mathematics instruction in their classrooms using the *Math Advantage* program (Harcourt Brace Publishing, 2000). The word problem-solving unit developed for this study supplanted the classroom mathematics lessons on 1 day in the 5-day week cycle. Both conditions included 20 instructional sessions that were implemented for 100 min at a time on 1 day during the week. The duration of instruction was approximately 25 hr and delivered across 18 weeks of the school year. Instructional sessions were held on only 1 day per week to accommodate the schedules of participating teachers at this school.

SBI. Students received instruction for solving one-step problems that involved two phases, problem schema and problem solution. *Problem schema* instruction used story problems that did not contain any unknown information to allow students to focus attention on identifying the problem schema (change, group, compare) and representing information in the story situation by using schematic diagrams. The emphasis on mapping the details of the story onto the schema diagram ensured that the student accurately represented the story problem in the diagram on the basis of the essential features of the problem schema.

During the *problem–solution* phase, students solved problems with unknowns. A four-step instructional procedure, FOPS (Find the problem type, Organize the information in the problem using the diagram, Plan to solve the problem, Solve the problem), was used to help anchor students' learning of the schema strategy in solving change, group,

TABLE 2. Comparison of SBI and GSI

SBI	GSI
Word problem-solving tasks • The number and type of addition and subtraction word problems were the same. Word problem-solving instruction • A teacher-mediated, guided practice-paired partner work-independent practice paradigm was used. Teacher-mediated instruction emphasized frequent teacher–student exchanges during problem solving. • Instructional sequence was as follows: change, group, compare, and two-step problems. (Either addition or subtraction operation was used to solve for the unknown quantity in word problems involving the three-problem schemata.)	Word problem-solving tasks • The number and type of addition and subtraction word problems were the same. Word problem-solving instruction • A teacher-mediated, guided practice-paired partner work-independent practice paradigm was employed. Teacher-mediated instruction emphasized frequent teacher–student exchanges during problem solving. • Instructional sequence was as follows: addition problems, subtraction problems, and two-step problems.

• Lessons addressed solving one- (3 units, with each unit comprising 10–11 lessons for a total of 32 lessons) and two-step problems (1 unit comprising 4 lessons). Students completed three review lessons following instruction of one-step problems and two mixed-practice review lessons following instruction of two-step problems.
• Instruction for solving one-step problems involved two phases, problem schema and problem solution.
  • *problem schema*—story problems did not contain any unknown information
  • *problem solution*—problems contained unknown values
• A four-step problem solving procedure, FOPS (Find the problem type, Organize the information in the problem using the diagram, Plan to solve the problem, Solve the problem) was used to anchor student learning in solving change, group, and compare problems.
• Instruction for two-step problems focused on chaining two schemata. A backward-chaining procedure was used to "actively search for the goal set (the unknown) and develop a solution plan from the goal set to the given information" (S. Goldman, 1989, p. 53).
	• Lessons addressed solving one-step and two-step word problems. Each unit on addition and subtraction one-step problems included 16 lessons (i.e., 32 total). Students completed one unit comprising four lessons on two-step problems, three review lessons following instruction of one-step problems, and two mixed-practice review lessons following instruction of two-step problems.
	• Instruction for solving one-step problems involved the use of several word problem-solving strategies (e.g., using objects, acting it out or drawing a diagram, choosing an operation or writing a number sentence, using data from a graph or table).
	• A four-step word problem-solving procedure (read and understand the problem, plan to solve the problem, solve the problem, and look back or check) was used to anchor students' learning in solving addition and subtraction word problems.
	• Instruction for solving two-step problems used the same four-step model and did not differ from solving one-step problems.

Note. SBI = schema-based instruction; GSI = general strategy instruction.

and compare problems. Instruction was designed to fade the schematic diagrams at the end of the instructional unit on each problem type. The fading procedure entailed replacing the schematic diagrams by diagrams jointly constructed by teachers and students (see Figure 2). It must be noted that these diagrams maintained the underlying problem structure and were shorthand styles of the original diagrams. We subsequently describe how SBI is used to solve change, group, and compare problems.

Change problems. Students first learned to identify change problems by using story situations such as "Christine had 18 tropical fish before she went to the pet store and bought 12 more fish. Now Christine has 30 tropical fish." Using the first step of FOPS (i.e., find the problem type), students identified the story situation as change, because an action occurred that increased the initial quantity of 18 tropical fish by 12 fish to result in an ending quantity of 30 tropical fish. For the next step (i.e., organize the information in the problem using the diagram), students used the change diagram (see Figure 1) to organize or represent the information. They identified and wrote the object identity or label (e.g., tropical fish) and the associated quantities for the three items of information (i.e., beginning, change, and ending) in the change diagram. Last, students used the completed diagram to summarize the information in the story and check the accuracy of the representation by transforming the information in the diagram into a number sentence ("18 + 12 = 30"). Students learned that a correct representation should lead to the correct number sentence.

During the problem–solution phase, SBI addressed the four steps of the FOPS procedure as students learned to solve for the unknown quantity in word problems such as "A squirrel made a pile of nuts. It carried away 55 nuts up to its nest. Now, there are 38 nuts in the pile. How many nuts were in the pile at the beginning?" Students were prompted to first identify and represent the problem (i.e., find and organize) using the change diagram much as in the problem schema instruction phase. The only difference was that

Linda walks her neighbors' dogs. On Monday, she walks 6 dogs. On Tuesday, she walks 3 more dogs than on Monday. How many dogs does she walk altogether on both days?

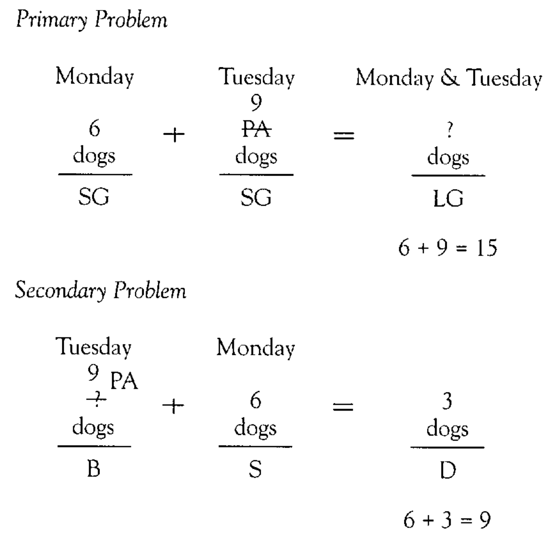


FIGURE 2. Solving a two-step word problem. SG = small group; LG = large group; PA = partial answer; B = bigger; S = smaller; D = difference.

students used a question mark ("?") to represent the unknown quantity in the diagram (see Figure 1). The next step (i.e., plan to solve the problem) had students select the appropriate operation and translate the information in the diagram into a number sentence. Instruction emphasized that when the change action caused an increase, the ending quantity represented the big number or whole. When the change action involved a decrease, the beginning quantity was the big number or whole. That is, students learned that they should add the parts if the big number or whole was unknown (e.g., "? – 55 = 38") and subtract to solve for the part if the big number or whole was known. In the final step (solve the problem), students solved for the unknown in the math sentence by using the operation identified in the previous step and checked not only the reasonableness of their answer but also the accuracy of both the representation and computation.

Group problems. Students first identified the problem type by using story situations such as "Ken and Art are selling candy bars at a fundraiser. Ken sold 27 candy bars. Art sold 43 candy bars. They sold 70 candy bars altogether." Using the first step in FOPS, they learned that the story represented a group problem, because it described a situation in which two small groups (candy bars that Ken sold, candy bars that Art sold) combined to form a large group (all the candy bars that they sold together). For the next step, students used the group diagram (see example diagram in Figure 1) to represent the information in the story by identifying the two small groups and the large group and writing the group names and the associated quantities in the diagram. Next, students summarized the information in the story by using the completed diagram and checked the accuracy of the representation by translating the information in the diagram into a number sentence ("27 + 43 = 70"). As in change stories, students checked the completed diagram by reviewing the information related to each set (i.e., the large group and the two small groups).

During the problem–solution phase, students solved word problems such as "Farmer Joe has 88 animals on his farm. He only has horses and goats. There are 49 horses on the farm. How many goats are on the farm?" Initially, students identified and represented the problem by using the group schematic diagram (see Figure 1) and then proceeded with the last steps in ways similar to those described for the change problem. When selecting the operation to solve for the unknown quantity in group problems, students learned that the large group is the big number or whole and that the small groups are the parts that make up the whole.

Compare problems. Students first identified the problem type as the compare problem by using story situations such as "Tina saw 6 movies this summer. She saw 6 fewer movies than Ruth. Ruth saw 12 movies." Students first approached the compare story situation as they would the change and group problems by identifying the story situation. This example problem is a compare problem type because it requires a comparison of the number of movies seen by Tina and Ruth. Students then used the compare diagram (see example diagram in Figure 1) to organize or represent the information by reading the comparison sentence ("Tina saw 6 fewer movies than Ruth") to identify the two sets compared in the story, determining the identity of the large (Ruth) and small (Tina) sets, labeling them in the diagram, and writing the difference amount in the diagram. Students then read the story to find the quantities associated with the two sets and wrote them in the diagram. Finally, students summarized the information in the story using the completed diagram and checked the accuracy of the representation by translating the information in the diagram into a number sentence ("12 – 6 = 6"). As in change and group stories, students checked the completed diagram by reviewing the information related to each set (i.e., big, small, and difference).

During the problem–solution instructional phase, students solved word problems such as "Craig saw a pine tree in the forest. Later, he saw a maple tree that was 9 feet tall. The maple tree was 5 feet shorter than the pine tree. How tall is the pine tree?" First, students identified and represented the problem using the compare schematic diagram (see Figure 1). Then, they proceeded in ways similar to those described for the change and group problems. When selecting the operation to solve for the unknown quantity in compare problems, students learned that the large or big set is the big number or whole and that the small set and difference are the parts that make up the whole.

Initially, only one type of word problem with the corresponding schema diagram was included on student worksheets following the instruction of that word problem type (e.g., change problem). After students learned how to solve change and group problems, word problems with both types were presented, prompting discussions of the similarities and differences between change and group problems. Later, when students had completed instruction for change, group, and compare problem types, worksheets included word problems with the three problem types.

Two-step word problems. Two-step word problems were taught using a backward chaining procedure that links the two schemata in these problems. It uses a top-down approach in which the learner is required to identify the overall or primary problem schema to be solved (Marshall, 1995). Figure 2 illustrates solving a two-step word problem using faded versions of the schematic diagrams. In the word problem about Linda, instruction focuses on the question about the number of dogs that were walked on both Monday and Tuesday and the surrounding context to indicate that the primary schema is a group problem. Much as in solving one-step problems, students used a "?" to represent the unknown quantity to be solved and wrote "PA" or "partial answer" for the other quantity in the primary schema faded diagram that was also unknown but could be determined by solving the secondary schema. Students then identified the secondary problem (i.e., compare) that must be solved to answer the primary problem. They reasoned that it is a compare problem, because it compares the number of dogs walked on Tuesday and the number of dogs walked on Monday. Next, students represented the information for the secondary problem and solved for the two unknowns (*PA* and *?*) in the two-step problem.

General Strategy Instruction (GSI). Students solved word problems using the following four-step problem-solving procedure based on Pólya's (1945/1990) model: (a) read and understand the problem, (b) plan to solve the problem, (c) solve the problem, and (d) look back or check. In addition, four word problem-solving strategies (i.e., using objects, acting it out or drawing a diagram, choosing an operation or writing a number sentence, and using data from a graph or table) commonly seen in third-grade mathematics textbooks were incorporated in the plan step of the problem-solving heuristic. During instructional sessions, the four-step problem-solving heuristic was displayed on large poster boards in the classrooms and served as a checklist to monitor application of the problem-solving procedure. At the beginning of each lesson, teachers either presented or reviewed the four problem-solving steps using example problems with teacher modeling.

The *understand* step was the first in the four-step problem-solving procedure used in GSI. The teacher and students first read the addition or subtraction problem. Next, the teacher asked facilitative questions (e.g., "What do you know about the squirrel and the piles of nuts?" "What must you solve for in the problem?") to ensure that students understood the problem. In the *plan* step, students were introduced to each one of the four strategies successively in subsequent lessons. For the *using objects* strategy, students had access to manipulatives (e.g., counters) and were prompted to use counters to represent the information in the word problem prior to solving it. When using the *acting it out or drawing a diagram* strategy, students were encouraged to either perform a short skit or draw a picture to represent the information in the problem. For example, teachers questioned students as follows: "How can we act out the problem or draw a picture of the problem to show or explain the problem?" When teaching the *choosing an operation or writing a number sentence* strategy, teachers guided students to choose either addition or subtraction to solve the word problem and assisted them in writing a number sentence (e.g., "34 + 14 = ?") to help them solve the problem. Finally, the *using data from a graph or table* strategy involved teachers' showing students how to use data from a graph or table as they planned to solve the problem.

During the *solve* step, teachers prompted students to apply the strategy or operation chosen in the plan step to solve the problem. For example, students counted the counters to add or subtract as they solved the problem or they constructed a number sentence and then solved the equation. The teacher also facilitated problem solution as follows: "Okay, now we can use the number sentence to show the answer to the word problem. What is the answer?" For the final step of the word problem-solving process, *look back* or *check*, teachers asked students to think about their answers, consider whether their answers made sense, and justify their answers.

Two-step problems were taught by using the same four-step heuristic and strategies used for solving one-step problems. However, students were made aware during two-step problem instruction that they could use the same problem-solving strategies that they learned for one-step problems to solve two-step addition and subtraction problems.

Measures and Data Collection

Two research assistants administered and scored the SAT-9 mathematics subtests, mathematical word problem-solving tests, and computation tests using scripted directions and answer keys. All data were collected in a whole-class arrangement.

Mathematics problem-solving and procedures subtests of the SAT-9. To determine whether the SBI and GSI groups were equivalent in mathematics knowledge, we administered the Abbreviated Battery of the SAT-9 mathematics test at the beginning of the study. The SAT-9 is a group-administered, multiple-choice, standardized test with national norms. The test comprises two mathematics subtests, SAT-9 MPS and SAT-9 MP, which assess mathematical content recommended by the NCTM (2000). The Problem Solving subtest includes 46 items that assess number theory, geometry, algebra, statistics, and probability. The Procedures subtest includes 30 computation items, 12 of which appear in context. Reliability (alpha coefficient) at third grade was .83 for SAT-9 MPS and .80 for SAT-9 MP.

Mathematical word problem-solving measure (WPS). To examine mathematics competence on third-grade addition and subtraction mathematical word problems, students completed an experimenter-designed WPS test prior to the intervention (pretest), at the end of the intervention (posttest), and 12 weeks after the end of the intervention (maintenance). The WPS test was developed to include 16 word problems that met the semantic criteria for change, combine (or group), or compare word problem types (Carpenter et al., 1983; Carpenter & Moser, 1984; Carpenter, Moser, & Bebout, 1988). Items for the WPS test were selected from commonly used third-grade mathematics textbooks and included two word problems with distracters. Students were required to apply simple (e.g., single-digit numbers) to complex computation skills (e.g., three- and four-digit numbers, regrouping) to solve the word problems. They were given 50 min to complete the same 16-item test (12 one-step and 4 two-step word problems) at pretest, posttest, and maintenance. Students were asked to show their complete work and to write the answer and label. For each item's scoring, we assigned 1 point for the correct number model and 1 point for the correct answer and label. The total possible score on the WPS test was 32 points. On this sample, Cronbach's alpha coefficients for the pretest, posttest, and maintenance test were .84, .89, and .83, respectively. Interscorer agreement computed on 20% of all three measures by two research assistants, who independently scored the protocols, was 0.99 for the pretest, 0.95 for the posttest, and 1.00 for the maintenance test.

Word Problem Solving Fluency Measure (WPS-F). To monitor student progress in solving word problems across the intervention phase of the study, an 8-item WPS-F test was administered once every 3 weeks. Students had 10 min to complete the problems. This test was similar to the WPS test with regard to problem types but differed in that it included fewer problems, had no distracters, and required applying addition and subtraction computation skills involving one- and two-digit numbers only. Six separate forms of the WPS-F measure were developed that included 6 one-step and 2 two-step problems and differed with respect to numbers, context, and position of problems, which were random. All possible combinations of problem types and operations were covered in two tests rather than in one test. To control for the difficulty of the tests, we designed odd-numbered probes and even-numbered tests to be parallel forms. Therefore, the aggregated score across two assessments (i.e., Tests 1 and 2, Tests 3 and 4, and Tests 5 and 6) was used in the analysis. All tests were administered after the initiation of the intervention, and the aggregated scores represented three occasions during the intervention. Directions for administering and scoring the tests were similar to those for the WPS test. On this sample, Cronbach's alpha coefficients were .84, .80, and .83 for Time 1, Time 2, and Time 3, respectively. Interscorer agreement on 20% of the six forms was 0.93 (Test 1), 0.99 (Test 2), 0.98 (Test 3), 0.96 (Test 4), 1.00 (Test 5), and 0.99 (Test 6), respectively.

Basic math computation fluency measure (L. S. Fuchs, Hamlett, & Fuchs, 1998). To examine the extent to which students were proficient on third-grade mathematics computation, we monitored them prior to and at the completion of the intervention using basic math computation probes. Students were required to complete 25 problems in 3 min. The maximum possible score on the computation probe was 43 correct digits. On this sample, Cronbach's alpha was .77 for each of the pretest and the posttest. Interscorer agreement was 0.99 for the pretest and 0.97 for the posttest.

Fidelity of Treatment

The nature of instruction for the two groups was standardized using scripted lessons specific to the treatment conditions (i.e., SBI and GSI). A list of 10 questions was developed that addressed salient instructional features from the scripts for each condition. Using these questions, two research assistants independently observed 100% of the instructional sessions and rated teachers in both conditions as *yes (the item was observed)* or *no (the item was not observed)*. Across independent observations, treatment fidelity was 98% (range = 90–100%) for the SBI group and 94% (range = 80–100%) for the GSI group.

Data Analysis

For all measures, the unit of analysis was each student's individual score. We did not model instructional groups within treatment conditions because of sample limitations. On the WPS and computation pretest measures, we conducted a separate one-way between-subjects ANOVA to examine initial group comparability. To investigate the influence of word problem-solving instruction on students' acquisition and maintenance of problem-solving skills following the intervention, we conducted a repeated measures analysis of covariance (ANCOVA) involving a one-way between-subjects and one-way within-subject factor, time (pretest vs. posttest vs. maintenance test) on the WPS scores. We specified the contrasts a priori as pretest-to-posttest and posttest-to-maintenance-test comparisons. Although the two groups did not differ on the SAT-9 MPS and SAT-9 MP tests at pretreatment, SAT-9 MP was used as a covariate because significant correlations were found at $p < .01$ for the SAT-9 MP and the WPS pretest ($r = .37$), posttest ($r = .30$), and maintenance test ($r = .31$).

To determine the impact of word problem-solving instruction on students' problem-solving performance immediately following the implementation of the intervention at Time 1, we conducted a one-way between-subjects ANCOVA, with the SAT-9 MP as a covariate. In addition, a repeated measures ANCOVA involving a one-way between-subjects and one-way within-subject factor, time (Time 1 vs. Time 2 vs. Time 3) was conducted on the WPS-F scores to examine progress over time. SAT-9 MP was used as a covariate, because it was significantly correlated at $p < .05$ with the word problem-solving fluency tests (r = .31, .37, and .36 with Time 1, Time 2, and Time 3, respectively).

To examine the influence of word problem-solving instruction on computation performance, we carried out a repeated measures ANCOVA involving a one-way between-subjects and one-way within-subject factor, time (pre vs. post), on the computation scores. Again, we used the SAT-9 MP as a covariate, because it was significantly correlated at $p < .01$ with the computation at both pretest ($r = .41$) and posttest ($r = .33$). ANCOVA was selected to reduce the probability of a Type II error, increase power by reducing the error variance, and control for variability in the SAT-9 MP (StatSoft, 1998). To estimate the practical significance of effects, we computed effect sizes (Cohen's d) by dividing the difference between the regressed adjusted means (i.e., adjusted for the covariate, SAT-9 MP) by the square root of the mean square error (Glass, McGaw, & Smith, 1981).

Results

Table 3 presents the mean scores and standard deviations on the WPS, WPS-F, and computation measures, by time and condition.

Pretreatment Comparability

Results indicated no statistically significant differences between groups on either WPS, $F(1, 58) = 0.07$, $p = .79$, or computation, $F(1, 58) = 0.01$, $p = .94$, indicating group equivalency before the beginning of the study.

Acquisition and Maintenance of Word Problem-Solving Performance as a Function of Treatment

Results on between-subjects effects showed that the main effect for group, $F(1, 57) = 0.05$, $p = .83$, was not statistically significant. However, the SAT-9 MP, $F(1, 57) = 9.61$, $p < .01$, was found to be a significant covariate. Tests of within-subject contrasts indicated a statistically significant main effect for time, $F(1, 57) = 10.12$, $p < .01$, on the pretest and posttest scores. Time at posttest when compared with time at pretest had a large effect size (.78). However, Time × Group interaction, $F(1, 57) = 0.69$, $p = .41$, and Time × SAT-9 MP-Score interaction, $F(1, 57) = 0.71$, $p = .40$, were not statistically significant. On the posttest and maintenance test scores, the main effect for time, $F(1, 57) = 0.64$, $p = .43$, was not statistically significant. In addition, Time × Group interaction, $F(1, 57) = 0.14$, $p = .7$, and Time × SAT-9 MP-Score interaction, $F(1, 57) = 0.04$, $p = .83$, were not statistically significant.

Progress on Word Problem Solving During the Intervention

Results of the analysis at Time 1 revealed a statistically significant difference between groups, $F(1, 57) = 13.07$, $p < .01$, favoring the SBI group. A large effect size of .94 was found for SBI when compared with GSI. In addition, results of the repeated measures between-subjects effects showed that the main effect for group, $F(1, 57) = 3.61$, $p < .05$, was statistically significant. A small effect size of .35 was found for SBI in comparison with GSI. The SAT-9 MP-Score interaction, $F(1, 57) = 11.67$, $p < .01$, was found to be a significant covariate. The main effect for time was not statistically significant, $F(1, 57) = 1.66$, $p > .05$. However, the Time × Group interaction, $F(1, 57) = 7.92$, $p < .01$, was statistically significant (see Figure 3). That is, the between-groups differences decreased over time.

Acquisition of Computation Performance as a Function of Treatment

Results revealed a statistically significant main effect for time, $F(1, 57) = 10.98$, $p < .01$. Time at posttest when

TABLE 3. Means, Standard Deviations, and Adjusted Means on Mathematics Tests, by Time and Condition

Measure	SBI			GSI		
	M	SD	M adj.	M	SD	M adj.
Word problem solving (WPS)						
Pretest	16.88	7.37	16.70	17.40	7.46	17.57
Posttest	22.52	5.76	22.39	21.70	7.98	21.83
Maintenance test	20.85	6.07	20.74	19.45	6.09	19.57
Word problem-solving fluency (WPS-F)						
Time 1	23.58	5.08	23.46	18.17	6.49	18.29
Time 2	24.13	6.09	24.02	22.03	4.41	22.15
Time 3	24.25	4.74	24.12	23.07	6.98	23.20
Computation						
Pretest	19.57	7.48	19.39	19.43	6.36	19.61
Posttest	26.27	8.55	26.10	25.67	7.54	25.83

Note. SBI = schema-based instruction; GSI = general strategy instruction; adj. = adjusted.

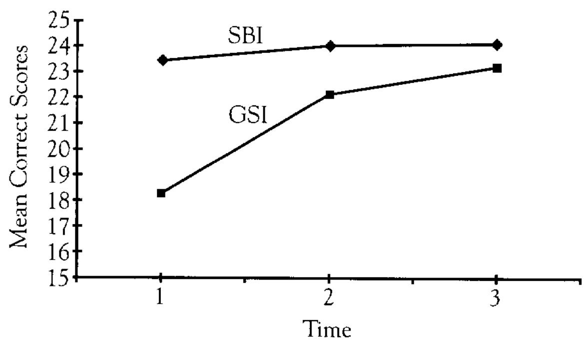


FIGURE 3. Performance on the WPS-F measure at Time 1, Time 2, and Time 3 during the intervention phase for the SBI and GSI conditions. WPS-F = word problem-solving fluency; SBI = schema-based instruction; GSI = general strategy instruction.

compared with time at pretest had a large effect size (.97). The main effect for group, $F(1, 57) = 0.000$, $p = .99$, and the Time × Group interaction, $F(1, 57) = 0.08$, $p = .78$, were not statistically significant. The SAT-9 MP-Score interaction, $F\ (1, 57) = 11.46$, $p < .001$, was a significant covariate. However, Time × SAT-9 MP-Score interaction, $F(1, 57) = 0.04$, $p = .84$, was not statistically significant.

Discussion

Is there a positive word problem-solving learning effect associated with SBI? The findings of the present study do not replicate the findings of Jitendra, Griffin, Haria, et al.'s (2007) study. That is, in the present study, the SBI condition did not lead to greater benefits in comparison with the GSI condition in the solution of word problems, as hypothesized. One explanation for this finding could be that the instructional format of 100 min per session masked the differential effects of the two instructional approaches over time, a possibility that was corroborated during the intervention phase and is described in the next section (Donovan & Radosevich, 1999). In fact, this type of massed presentation of material and student involvement is not common in elementary classrooms, and its benefits with regard to memory and sustaining student interest are questionable (Seabrook, Brown, & Solity, 2005).

At the same time, students in these inclusive third-grade classrooms improved in their word problem-solving performance from pretest to posttest (effect size = .78), regardless of whether they received SBI or GSI. This finding supports the notion that both SBI and GSI produced more accurate solutions on the posttests, an effect that is consistent with the tenets of a responsiveness-to-intervention (RTI) model (D. Fuchs & Fuchs, 2006). As public schools move from traditional service delivery models to more inclusive education in which students of varying abilities are taught in one classroom, it is important for educators and researchers to ascertain whether all students are benefiting from instruction. Thus, the present study is significant because it appears that both SBI and GSI approaches can substantially foster word problem-solving learning. However, the field needs more research that focuses on the context conditions under which SBI and GSI are effective. That is, the advantage of SBI over GSI may be more evident when instructional conditions reflect actual classroom practices (e.g., scheduling), as in Jitendra, Griffin, Haria, et al.'s (2007) study with low-achieving students.

Another notable finding related to the impact of SBI and GSI over time is that third-grade students in both conditions maintained the positive learning effect 12 weeks later. Although researchers have evaluated the transfer effects of SBI (e.g., L. S. Fuchs et al., 2006; Lewis, 1989), few studies (e.g., Jitendra, Griffin, Haria, et al., 2007; Jitendra et al., 1998) have explored the influence of SBI on students' ability to maintain their learning. Consequently, by including a maintenance measure and doubling the time (from 6 to 12 weeks) between posttesting and delayed posttesting relative to previous research (Jitendra, Griffin, Haria, et al.), the present study contributes to the growing support for strategy instruction to maintain students' mathematics problem solving long after the instructional intervention period has ended.

Is there a differential impact between the use of SBI and the use of GSI on word problem-solving performance immediately following instruction and across time during the intervention? According to the results of this study, the SBI approach resulted in a statistically significant effect (effect size = .94) on WPS-F scores at Time 1 during the intervention when compared to the GSI approach. It is notable that the advantage of the SBI over GSI was achieved relatively quickly, considering that the time elapsed for learning was only 6 weeks into the intervention. In fact, the scores for the SBI group at Time 1 were comparable to the posttest scores. This is a particularly important accomplishment and could be attributed to the emphasis in SBI on the structure of a problem situation and the connections made between the mathematical operations of addition and subtraction to the problem situation. However, the analysis indicated that between-group differences at Time 1 decreased over time. Again, perhaps the earlier explanation of the less than optimal instructional format of 100 min per session accounted for this finding. A visual examination of the word problem-solving progress data revealed that the SBI group as a whole maintained the same high level of performance over time during the intervention (see Figure 3). In contrast, the GSI group showed growth from Time 1 to Time 2 and then maintained that performance until the end of the intervention period.

Does word problem-solving instruction affect mathematics computation learning? Given the role that word problems play in the development of number operations (Van de Walle, 2007), we also designed the present study to examine the influence of word problem-solving instruction on computational skills. The findings of the study support the hypothesis that students in both conditions (SBI and GSI) would improve their computational skills. Word problem-solving strategy instruction had a positive effect on students' computation performance from pretest to posttest (effect size = 0.97). The NCTM's (2000) call for a shift in emphasis from procedural knowledge to conceptual understanding in mathematics instruction cannot be ignored. However, the link between involvement in strategic word problem-solving instruction and subsequent improvement in scores on the computation measure is an important outcome for students in both conditions in this study. Approaches that focus on intensive, rote memorization of math facts may not be optimal or efficient. It is encouraging that word problem-solving instruction (both SBI and GSI) not only improved performance in solving word problems in this study but also positively influenced the computation fluency of all students in these inclusive third-grade classrooms. This finding suggests that high-quality word problem-solving instruction may be an effective instructional option in heterogeneous elementary classrooms to improve students' understanding of mathematics word problem solving and their computation accuracy.

Limitations

Several limitations of the study require cautious interpretation of the findings. First, we lacked a true control group, and that lack threatens internal validity. That is, although regression to the mean is a possible explanation for the effect for time, inclusion of a true control condition is essential to conclusively attribute the gains over time to either intervention condition. At the same time, previous researchers have revealed positive effects for SBI in comparison with control group conditions that are guided by typical mathematics instruction in basal textbook instruction (e.g., L. S. Fuchs et al., 2006; Jitendra et al., 1998; Xin et al., 2005).

Second, the distribution of time in the study (i.e., 100-min session per week) did not reflect typical classroom practice. Although the teachers assumed responsibility for the unusual instructional schedule, it is conceivable that they did not consider the effect of using massed practice instead of spaced practice in learning (e.g., Donovan & Radosevich, 1999; Rohrer & Taylor, 2006). We are convinced that this arrangement could have influenced students' ability to internalize and retain SBI. Teaching SBI over shorter, more frequent lessons could have allowed students to acquire the important features of SBI and use the instruction to improve their problem-solving abilities more than they did in the present study. Finally, although reading comprehension is an important factor contributing to students' word problem-solving performance (Zentall & Ferkis, 1993), we did not control for students' initial reading levels in the present study. Thus, it is not clear to what extent reading comprehension skills contributed to the present findings.

Implications and Directions for Future Research

The most important message of this article is that sound strategy instruction can produce a positive learning effect. Overall, the use of SBI or GSI is recommended, because both approaches were equally effective in improving third-grade students' ability to solve both word and computational problems. Thus, this research shows that children with a variety of learning strengths and challenges can be successfully maintained in general education classrooms if teachers provide adequate opportunities for problem solving and use effective strategies, and this finding is consistent with a RTI approach (D. Fuchs & Fuchs, 2006).

However, the present work also generated a number of new research questions that may be addressed through further investigations. The field needs future researchers to consider the lack of effects for SBI in comparison with GSI in the present study. One way of doing this might be to replicate this study by ensuring that the 100 min of instructional time for word problem-solving instruction is distributed across the week to address concerns associated with massed practice in learning. Furthermore, examining the differential effects of instruction on students performing at various achievement levels (e.g., low, average, and high) may help to determine whether SBI supports some learners better than others or all levels comparably. Unfortunately, the small sample size prevented such comparisons in this study. Another fruitful goal for future research would be to explore alternative conditions under which the SBI approach may be effective. For example, examining the effects of SBI with students of different grade levels (e.g., fourth- to eighth-grade students) and applying other mathematical operations (i.e., multiplication and division) may produce results that further generalize the positive effects of SBI. In sum, the present findings suggest that the use of high-quality strategy instruction in mathematics has the potential to improve the word problem-solving performance and computation abilities of diverse learners in third-grade elementary school classrooms.

ACKNOWLEDGMENTS

The present research was supported by U.S. Department of Education, Office of Special Education Programs, grant H324D010024. The authors are grateful to Dr. Russell Gersten for his expert advice throughout the project. The authors thank the administrators, teachers, and students at P. K. Yonge Developmental Research School, without whose patience and perseverance this study would not have been possible.

REFERENCES

Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *Elementary School Journal, 103*, 51–73.

Briars, D. J., & Larkin, J. H. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction, 1*, 245–296.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Hiebert, J., & Moser, J. M. (1983). The effect of instruction on children's solutions of addition and subtraction word problems. *Educational Studies in Mathematics, 14*, 55–72.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15*, 179–202.

Carpenter, T. P., Moser, J. M., & Bebout, H. C. (1988). Representation of addition and subtraction word problems. *Journal for Research in Mathematics Education, 19*, 345–357.

Chen, Z. (1999). Schema induction in children's analogical problem solving. *Journal of Educational Psychology, 91*, 703–715.

Donovan, J. J., & Radosevich, D. J. (1999). A meta-analytic review of the distribution of practice effect: Now you see it, now you don't. *Journal of Applied Psychology, 84*, 795–805.

Fuchs, D., & Fuchs, L. S. (2006). Introduction to response to intervention: What, why, and how valid is it? *Reading Research Quarterly, 41*, 93–99.

Fuchs, L. S., Fuchs, D., Finelli, R., Courey, S. J., & Hamlett, C. L. (2004). Expanding schema-based transfer instruction to help third graders solve real-life mathematical problems. *American Educational Research Journal, 41*, 419–445.

Fuchs, L. S., Fuchs, D., Finelli, R., Courey, S. J., Hamlett, C. L., Sones, E. M., et al. (2006). Teaching third graders about real-life mathematical problem solving: A randomized controlled study. *Elementary School Journal, 106*, 293–312.

Fuchs, L. S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C. L., Owen, R., et al. (2003a). Explicitly teaching for transfer: Effects on third-grade students' mathematical problem solving. *Journal of Educational Psychology, 95*, 293–304.

Fuchs, L. S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C. L., Owen, R., et al. (2003b). Enhancing third-grade students' mathematical problem solving with self-regulated learning strategies. *Journal of Educational Psychology, 95*, 306–315.

Fuchs, L. S., Fuchs, D., Yazdian, L., & Powell, S. R. (2002). Enhancing first-grade children's mathematical development with peer-assisted learning strategies. *School Psychology Review, 31*, 569–583.

Fuchs, L. S., Hamlett, C. L., & Fuchs, D. (1998). *Monitoring basic skills progress: Basic math computational manual* (2nd ed.). Austin, TX: Pro-Ed.

Fuson, K. C., Carroll, W. M., & Landis, J. (1996). Levels in conceptualizing and solving addition and subtraction compare word problems. *Cognition and Instruction, 14*, 345–371.

Fuson, K. C., & Willis, G. B. (1989). Second graders' use of schematic drawings in solving addition and subtraction word problems. *Journal of Educational Psychology, 81*, 514–520.

García, A. I., Jiménez, J. E., & Hess, S. (2006). Solving arithmetic word problems: An analysis of classification as a function of difficulty in children with and without arithmetic LD. *Journal of Learning Disabilities, 39*, 270–281.

Glass, G., McGaw, B., & Smith, M. (1981). *Meta-analysis and social research*. Thousand Oaks, CA: Sage.

Goldman, S. (1989). Strategy instruction in mathematics. *Learning Disability Quarterly, 12*, 43–55.

Goldsmith, L. T., & Mark, J. (1999). What is a standards-based mathematics curriculum? *Educational Leadership, 57*, 40–44.

Harcourt Brace & Company. (1996). *Stanford achievement tests* (9th ed.). San Antonio, TX: Harcourt Brace Jovanovich.

Harcourt Brace Publishing. (2000). *Math Advantage Grade 3*. Orlando, FL: Harcourt School Publishers.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology, 91*, 684–689.

Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Education Psychology, 87*, 18–32.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., et al. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25*(4), 12–21.

Hutchinson, N. L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disabilities Quarterly, 16*, 34–63.

Jitendra, A., DiPipi, C. M., & Perron-Jones, N. (2002). An exploratory study of schema-based word problem solving instruction for middle school students with learning disabilities: An emphasis on conceptual and procedural understanding. *Journal of Special Education, 36*, 23–38.

Jitendra, A., Griffin, C., Deatline-Buchman, A., DiPipi-Hoy, C., Sczesniak, E., Sokol, N., et al. (2005). Adherence to mathematics professional standards and instructional design criteria for problem-solving in mathematics. *Exceptional Children, 71*, 319–337.

Jitendra, A. K., Griffin, C., Deatline-Buchman, A., & Sczesniak, E. (2007). Mathematical word problem-solving in third grade classrooms: Lessons learned from design experiments. *The Journal of Educational Research, 100*, 283–302.

Jitendra, A. K., Griffin, C. C., Haria, P., Leh, J., Adams, A., & Kaduvettoor, A. (2007). A comparison of single and multiple strategy instruction on third grade students' mathematical problem solving. *Journal of Educational Psychology, 99*, 115–127.

Jitendra, A. K., Griffin, C. C., McGoey, K., Gardill, M. C., Bhat, P., & Riley, T. (1998). Effects of mathematical word problem solving by students at risk or with mild disabilities. *The Journal of Educational Research, 91*, 345–355.

Jitendra, A., & Hoff, K. (1996). The effects of schema-based instruction on mathematical word problem solving performance of students with learning disabilities. *Journal of Learning Disabilities, 29*, 422–431.

Jitendra, A. K., Hoff, K., & Beck, M. M. (1999). Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. *Remedial and Special Education, 20*, 50–64.

Jonassen, D. H. (2003). Designing research-based instruction for story problems. *Educational Psychology Review, 15*, 267–296.

Kalyuga, S., Chandler, P., Tuovinen, J., & Sweller, J. (2001). When problem solving is superior to studying worked examples. *Journal of Educational Psychology, 93*, 579–588.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review, 92*, 109–129.

Kroesbergen, E. H., Van Luit, J. E. H., & Maas, C. J. M. (2004). Effectiveness of explicit and constructivist mathematics instruction for low-achieving students in the Netherlands. *Elementary School Journal, 104*, 223–249.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem-solving behavior. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving* (pp. 75–88). New York: Springer-Verlag.

Lewis, A. B. (1989). Training students to represent arithmetic word problems. *Journal of Educational Psychology, 8*, 521–531.

Lopez-Real, F. (2006). A new look at a Polya problem. *Mathematics Teaching, 196*, 12–15.

Lucangeli, D., Tressoldi, P. E., & Cendron, M. (1998). Cognitive and metacognitive abilities involved in the solution of mathematical word problems: Validation of a comprehensive model. *Contemporary Educational Psychology, 23*, 257–275.

Marshall, S. P. (1995). *Schemas in problem solving*. New York: Cambridge University Press.

Mayer, R. E. (1999). *The promise of educational psychology: Vol. 1. Learning in the content areas*. Upper Saddle River, NJ: Merrill Prentice Hall.

Mayer, R. E., & Hegarty, M. (1996). The process of understanding mathematics problems. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking*. Mahwah, NJ: Erlbaum.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics*. Reston, VA: Author.

National Research Council (NRC). (2001). *Adding it up: Helping children learn mathematics*. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), A report from the NRC. Washington, DC: National Academies Press.

No Child Left Behind Act of 2001, 20 U.S.C. § 6301 *et seq*.

Pawley, D., Ayres, P., Cooper, M., & Sweller, J. (2005). Translating words into equations: A cognitive load theory approach. *Educational Psychology, 25*, 75–97.

Pólya, G. (1990). *How to solve it*. London: Penguin. (Originally published in 1945)

Pressley, M., & Hilden, K. (2006). Cognitive strategies. In D. Kuhn, R. S. Siegler, W. Damon, & R. M. Lerner (Eds.), *Handbook of child psychology:*

Vol. 2. Cognition, perception, and language (6th ed.; pp. 511–556). Hoboken, NJ: Wiley.

Quilici, J. L., & Mayer, R. E. (1996). Role of examples in how students learn to categorize statistics word problems. *Journal of Educational Psychology, 88*, 144–161.

Riley, M. S., Greeno, J. G., & Heller, J. I. (1983). Development of children's problem-solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153–196). New York: Academic Press.

Rohrer, D., & Taylor, K. (2006). The effects of overlearning and distributed practice on the retention of mathematics knowledge. *Applied Cognitive Psychology, 20*, 1209–1224.

Romberg, T. A., Carpenter, T. P., & Kwako, J. (2005). Standards-based reform and teaching for understanding. In T. A. Romberg, T. P. Carpenter, & F. Dremock (Eds.), *Understanding mathematics and science matters* (pp. 3–26). Mahwah, NJ: Erlbaum.

Schurter, W. A. (2002). Comprehension monitoring: An aid to mathematical problem solving. *Journal of Developmental Education*, 26(2), 22–33.

Seabrook, R., Brown, G. D. A., & Solity, J. E. (2005). Distributed and massed practice: From laboratory to classroom. *Applied Cognitive Psychology, 19*, 107–122.

Siegler, R. S. (2005). Children's learning. *American Psychologist, 60*, 769–778.

StatSoft (1998). *Analysis of Covariance (ANCOVA)*. Retrieved March 19, 2005, from http://sunsite.univie.ac.at/textbooks/statistics/stanman.html#analysis

Stylianou, D. A., & Silver, E. A. (2004). The role of visual representations in advanced mathematical problem solving: An examination of expert-novice similarities and differences. *Mathematical Thinking and Learning, 6*, 353–387.

Sweller, J., & Low, R. (1992). Some cognitive factors relevant to mathematics instruction. *Mathematics Education Research Journal, 4*, 83–94.

Sweller, J., Van Merrienboer, J. J. G., & Paas, F. G. W. C. (1998). Cognitive architecture and instructional design. *Educational Psychology Review, 10*, 251.

Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2005). Simple addition strategies in a first-grade class with multiple strategy instruction. *Cognition and Instruction, 23*, 1–21.

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Boston: Allyn and Bacon.

Van Garderen, D., & Montague, M. (2003). Visual-spatial representation, mathematical problem solving, and students of varying abilities. *Learning Disabilities Research & Practice, 18*, 246–254.

Vergnaud, G. (1997). The nature of mathematical concepts. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 5–28). Hove, England: Psychology Press.

Willis, G. R., & Fuson, K. C. (1988). Teaching children to use schematic drawings to solve addition and subtraction word problems. *Journal of Educational Psychology, 80*, 192–201.

Woodward, J. (2006). Making reform-based mathematics work for academically low-achieving middle school students. In M. Montague & A. K. Jitendra (eds.), *Teaching mathematics to middle school students with learning difficulties*. New York: Guilford.

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem solving instruction on middle school students with learning problems. *Journal of Special Education, 39*, 181–192.

Zawaiza, T. B. W., & Gerber, M. M. (1993). Effects of explicit instruction on community college students with learning disabilities. *Learning Disabilities Quarterly, 16*, 64–79.

Zentall, S. S., & Ferkis, M. A. (1993). Mathematical problem solving for youth with ADHD, with and without learning disabilities. *Learning Disabilities Quarterly, 16*, 6–18.